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UNITARY ROOT MUSIC AND UNITARY MUSIC WITH REAL-VALUED RANK REVEALING TRIANGULAR FACTORIZATION (Postprint)

Nizar Tayem and Mort Naraghi-Pour

Louisiana State University
Department of Electrical and Computer Engineering

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14. ABSTRACT
This paper presents two methods to estimate the two-dimensional (2-D) direction of arrival (DOAs) for coherent and non-coherent sources. The proposed methods have many advantages over existing schemes. First, they construct the data from a single snapshot in a Toeplitz form, whose rank is directly related to the DOA of signals, whether the signals are coherent or not; hence, the algorithm does not require any forward/backward spatial smoothing. Second, the two proposed methods can rapidly estimate the 2-D DOAs of incident signals without requiring singular value decomposition (SVD) or eigenvalue decomposition (EVD), even in the case of coherent signals and a single snapshot. The two methods are: (1) orthogonal projection real-valued rank revealing QR factorization (OP-RRRQR), and (2) orthogonal projection real-valued rank revealing LU factorization (OP-RRRLU). The proposed methods reduce the computational complexity and the cost at least by a factor of four by applying a unitary transformation, to the complex Toeplitz form to real data without forming the covariance matrix. The proposed algorithms employ the unitary root MUSIC and unitary MUSIC using cross array configuration to estimate the 2-D DOA azimuth and elevation angles without using the extensive 2-D MUSIC search. Hence, the proposed algorithm can reduce the computational load and cost significantly and can be applied to faster real-time radar/sonar and commercial wireless systems. The simulation results show that the proposed algorithm can efficiently estimate the 2-D DOAs from different sources.

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Unitary Root MUSIC and Unitary MUSIC with Real-Valued Rank Revealing Triangular Factorization[1]

Nizar Tayem and Mort Naraghi-Pour

Department of Electrical and Computer Engineering

Louisiana Sate University, Baton Rouge, LA 70803

Tel: (225) 578-5551, Fax (225) 978-5200, Email: mort@ece.lsu.edu

Abstract-This paper presents two methods to estimate the two dimensional (2-D) direction of arrival (DOA) for coherent and non-coherent sources. The proposed methods have many advantages over existing schemes. First, they construct the data from a single snapshot in a Toeplitz form, whose rank is directly related to the DOA of signals, whether the signals are coherent or not; hence, the algorithm does not require any forward/backward spatial smoothing. Second, the two proposed methods can rapidly estimate the 2-D DOAs of incident signals without requiring singular value decomposition (SVD) or eigenvalue decomposition (EVD), even in the case of coherent signals and a single snapshot. The two methods are: (1) orthogonal projection real-valued rank revealing QR factorization (OP-RRRQR), and (2) orthogonal projection real-valued rank revealing LU factorization (OP-RRRLU). The proposed methods reduce computational complexity and the cost at least by a factor of four by applying a unitary transformation, to the complex Toeplitz form to real data without forming the covariance matrix. The proposed algorithms employ the unitary root MUSIC and unitary MUSIC using cross array configuration to estimate the 2-D DOA azimuth and elevation angles without using the extensive 2-D MUSIC search. Hence, they can reduce the computational load and cost significantly and can be applied in real-time radar/sonar and commercial wireless systems. The simulation results show that the proposed algorithms can efficiently estimate the 2-D DOAs from different sources.

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I. INTRODUCTION

The problem of estimating the two-dimensional (2-D) direction of arrival (DOA), azimuth and elevation angles, for incident signals on an antenna array has received considerable attention in the field of array signal processing [1-13]. This problem has applications in many fields including radar, sonar, radio astronomy, seismic data processing, and mobile communication systems. Existing algorithms employ either eigenvalue decomposition (EVD) of the sample covariance matrix or singular value decomposition (SVD) of the data matrix to estimate the signal and noise space, and are computationally extensive and time consuming, especially when the number of antenna array elements N is larger than the number of incident signals. Furthermore, many of these methods require a two dimensional search, complex pair-matching of the azimuth and elevation angle, have an estimation failure problem when the elevation angles are between 70° and 90°, need a large number of samples to estimate the covariance matrix, and fail when the sources are highly correlated or completely coherent.

The 2-D DOA estimation problem in [1] requires an exhaustive 2-D peak search through all possible steering vectors. In addition, the sources must be non-coherent and a large number of snapshots are required to estimate the covariance matrix. In [2-5] the authors proposed a 2-D DOA estimation method using a different array configuration. While this method does not require a 2-D peak search, it has other drawbacks: (1) it requires a pair-matching between the 2-D azimuth and elevation angle estimations, and (2) estimation fails when the elevation angle is between 70° and 90° or when the signal-to-noise ratio (SNR) is Low.

The methods in [1-4, 6-13] require a large number of snapshots to estimate the covariance matrix. This covariance matrix will have a Toeplitz structure if the incident sources are uncorrelated and statistically stationary. However, if the incident sources are coherent (e.g., in a multipath environment), the covariance matrix is not Toeplitz. In order to obtain the Toeplitz structure, the preprocessing spatial smoothing technique [14-16] has been introduced in order to destroy the cross correlation between directional components; however, smoothing technique requiresa large number of snapshots and averaging the covariance matrices which increase the computational load and the complexity.

In this paper, we present a new approach to solve the 2-D DOA estimation problem. There are five main underlying ideas behind this approach. First, the proposed algorithm preserves the Toeplitz structure by mapping the $(2N+1)\times 1$ data vector from a single snapshot into an $(N+1)\times(N+1)$ Toeplitz matrix whose rank is related to the DOA of the incoming signals independent of whether the sources are coherent or not. Hence, the proposed method does not use forward/backward spatial smoothing resulting in a reduction in the computational complexity and cost. Second, the proposed method uses only a single snapshot of the received signals to estimate the DOA of the incident

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sources. This reduces the computational load drastically and makes the proposed method a good candidate for real-time implementation. Third, to reduce the computational complexity and cost further, we employ the unitary transformation in [17] to convert the complex-valued Toeplitz data to real-valued data. This reduces the processing time by almost a factor of four, since the cost of complex multiplication is four times that of real multiplication. Fourth, the two proposed methods can rapidly estimate the 2-D DOAs of incident signals without requiring SVD or EVD, even in the case of coherent signals and a single snapshot. The two methods are: (1) orthogonal projection real-valued rank revealing QR factorization (OP-RRRQR) and (2) orthogonal projection real-valued rank revealing LU factorization (OP-RRRLU). The rank revealing QR factorization [18-20] is precisely used to compute the subspace information and effectively update the signal information that can be used to track a moving source. The QR factorization is also applied to estimate the DOAs in the non-stationary environment of tracking moving sources. Fifth, we propose a cross array configuration that consists of two centro-symmetric uniform linear arrays in the z-x plane; then, the proposed unitary root MUSIC is applied to the array in the z axis to estimate the elevation angle. Subsequently, the unitary MUSIC method is used to estimate the azimuth by the use of the estimated elevation angle for each source. Finally, the proposed methods do not require any pair-matching between azimuth and elevation, they do not suffer from estimation failure, and avoid the 2-D search peak.

The rest of this paper is organized as follows. Section II presents the proposed 2-D DOA methods. Section III shows simulation results, and in Section IV we draw our conclusions.

II. SYSTEM MODEL

A-Orthogonal projection with real-valued rank-revealing QR factorization

Figure 1 shows the proposed array configuration consisting of two symmetric uniform linear arrays (ULA) with interspacing d equal to a half wavelength of incident signals. We assume that all the incident sources have the same carrier frequency. Each uniform linear array in Figure 1 consists of $2N$ elements, and the element placed at the origin is numbered 0 for reference purposes. One array is placed on the z axis and the other on the x axis. Suppose that there are K narrow band sources where the k-th source has an elevation angle θ_k and an azimuth angle φ_k, $k=1,\ldots,K$.

Step 1. Estimation of Elevation Angle θ_k

For a given snapshot t, the output signal from the k-th element on the z axis is given by

$$z_k(t)=\sum_{i=0}^{K} s_i(t)\, e^{-j(2\pi/\lambda)d\cos\theta_i}+n_k(t) \qquad (1)$$

where $s_i(t)$ is the signal from the i-th incident source, and $n_k(t)$ is the noise at the k-th element.

If we use the element at the center of the array as a reference point, then the $(2N+1)\times 1$ output vector from the $2N+1$ antenna elements placed on the z axis can be written as

$$Z(t)=\begin{bmatrix} z_{-N}(t)\\ \vdots \\ z_0(t)\\ \vdots \\ z_N(t)\end{bmatrix}=A(\theta)\;S(t)+N(t) \qquad (2)$$

where

$$\mathrm{A}(\theta)=\left[\mathbf{a}(\theta_1)\quad \mathbf{a}(\theta_2)\quad \ldots\quad \mathbf{a}(\theta_K)\right] \qquad (3)$$

is the $(2N+1)\times K$ array response matrix,

where

$$a(\theta_k)=\left[\left(u_k^*\right)^N\quad \cdots\quad 1\quad \cdots\quad u_k^N\right]^T \qquad (4)$$

is the corresponding $(2N+1)\times 1$ array response vector, with

$$u_k=\exp\left(-j2\pi\, d\cos\left(\theta_k\right)/\lambda\right), \qquad (5)$$

where $S(t)$ is the vector of received signals

$$S(t)=\left[s_1(t)\quad s_2(t)\quad \ldots\quad s_K(t)\right]^T \qquad (6)$$

and

$$N(t)=\left[\left(n_{-N}(t)\quad \cdots\quad n_0(t)\quad \cdots\quad n_N(t)\right)\right]^T \qquad (7)$$

where $N(t)$ is the $(2N+1)\times 1$ noise vector. Herein, the superscripts T and $*$ denote the transpose and conjugate operations, respectively.

In the proposed method, we map the output data vector $Z(t)$ with dimension $(2N+1)\times 1$ to a Toeplitz Hermition data matrix with dimension $(N+1)\times(N+1)$. The advantage of introducing the Toeplitz Hermition data matrix is that it has a rank that is related to the DOA of the sources independent of whether the sources are coherent or not. This matrix, denoted Y, is given by

$$Y=\begin{bmatrix} z_0 & z_{-1} & z_{-2} & \cdots & z_{-N}\\ z_1 & z_0 & z_{-1} & \cdots & z_{-(N-1)}\\ z_2 & z_1 & z_0 & \cdots & z_{-(N-2)}\\ \vdots & \vdots & \vdots & \ddots & \\ z_N & z_{N-1} & z_{N-2} & \cdots & z_0\end{bmatrix} \qquad (8)$$

where we have dropped the time index *t*.

Assuming a noise free case the Hermitian Toplitz data matrix in (8) can be rewritten in terms of $B(\theta)$, Φ_1^*, and S as follows

$$Y=\left[B(\theta)S \quad B(\theta)\Phi_1^*(\theta)S \quad \cdots \quad B(\theta)\left(\Phi_1^*(\theta)\right)^N S\right] \quad (9)$$

where

$$B(\theta)=\left[\mathbf{b}(\theta_1) \quad \mathbf{b}(\theta_2) \quad \ldots \quad \mathbf{b}(\theta_K)\right] \quad (10)$$

is the $((N+1)\times K)$ array response matrix where

$$\mathbf{b}(\theta_k)=\left[1 \quad u_k \quad \cdots \quad u_k^N\right]^T \quad (11)$$

and

$$\Phi_1^*=\begin{bmatrix} u_1^* & 0 & \cdots & 0 \\ 0 & u_2^* & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & u_K^* \end{bmatrix}=diag(u_1^* \quad u_2^* \quad \cdots \quad u_K^*) \quad (12)$$

The square Hermitian Toeplitz in (10) can be decomposed as follows

$$Y=WDW^H \quad (13)$$

where

$$W=\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ u_1 & u_2 & u_3 & \cdots & u_K \\ u_1^2 & u_2^2 & u_3^2 & \cdots & u_K^2 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_1^N & u_2^N & u_3^N & \cdots & u_K^N \end{bmatrix} \quad (14)$$

and

$$D=\begin{bmatrix} s_1 & 0 & \cdots & 0 \\ 0 & s_2 & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & s_K \end{bmatrix} \quad (15)$$

Since $Y=WDW^H$ and the diagonal signal matrix D is of full rank, the rank of Y is the same as that of W. The matrix W in (14) has the structure of an $(N+1)\times K$ Vandermonde matrix. The rank of W is equal to the Minimum of $(N+1)$ and K; hence, rank of W is equal to K. This means that the rank of the Hermition Toeplitz data matrix is equal to the number of DOAs of the sources whether the sources are coherent or not. Therefore, all the incident sources can be detected even if the sources are not coherent because the Toeplitz data matrix structure will be preserved in both scenarios (i.e., coherent and non-coherent sources).

If W matrix in (13) is the same as the collection of array response vectors from different directions in $B(\theta)$ in (11), then equation (13) can be rewritten as

$$Y=B(\theta)DB(\theta)^H \quad (16)$$

Unitary transformation Method for Toeplitz data

Let J represent the exchange matrix (i.e., 1's on the antidiagonal and 0's elsewhere) as

$$J=\begin{pmatrix} 0 & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & 0 \end{pmatrix} \quad (17)$$

Note that $J=J^H=J^{-1}$

Toeplitz matrices are *persymmetric*, meaningthat they are symmetric about their southwest-northeast diagonal. For such a matrix P

$$JP^TJ=P$$

(18)

Let U be an MXM matrix defined as

$$G=\frac{1}{\sqrt{2}}\begin{bmatrix} I & J \\ jJ & -jI \end{bmatrix} \quad (19)$$

where J and I are exchange and identity matrices, respectively, with dimensions $(M/2)\times(M/2)$, and G is a unitary matrix which satisfies

$$G^*J=G. \quad (20)$$

Equation (19) holds for an M even. In the case where M is odd, the unitary matrix G can be written as

$$G=\frac{1}{\sqrt{2}}\begin{bmatrix} I & 0 & J \\ \underline{0} & \sqrt{2} & \underline{0} \\ jJ & 0 & -jI \end{bmatrix} \quad (21)$$

where I and J have dimensions of $((M-1)/2)\times((M-1)/2)$ and $\underline{0}=[0,0,\ldots,0]$.

Construct the output data of the proposed method as follows

$$\Psi=Y+JY^*J \quad (22)$$

If we pre-multiply the construct data in (22) by G, post-multiply by G^H and take the conjugate, we get

$$\left(G\Psi G^H\right)^*=\left[G\left(Y+JY^*J\right)G^H\right]^*$$

$$=\left(G(Y+Y^H)G^H\right)^*=G^*Y^*G^T+G^*Y^TG^T \quad (23)$$

Note that because Y is a Toeplitz matrix then $JY^*J=Y^H$. Using $JJ=I$, we can rewrite (23) as follows

$$\left(G\Psi U^H\right)^*=G^*JJY^*JJG^T+G^*JJY^TJJG^T$$

$$=G^*J(JY^*J)JG^T+G^*J\left(JY^TJ\right)JG^T$$

$$=G^*J(Y^H+Y)JG^T \quad (24)$$

If we substitute (20) for (24), we get

$$\left(G\Psi G^H\right)^*=G\left(Y+Y^H\right)G^H$$

$$=G\Psi G^H \quad (25)$$

Therefore, $G\Psi G^H$ is a *real-valued* matrix; the decomposition of $G\Psi G^H$ requires only a real computation which means the computational load and cost will reduce significantly without effecting the accuracy of the DOAs.

The proposed method uses the orthogonal projection real-valued rank revealing QR factorization to estimate the 2-D DOA elevation and azimuth angle from coherent /non-coherent sources by using the real data matrix in (25). The necessary information about the noise subspace or the signal subspace can be extracted using the rank revealing QR factorization [18-20]. One of the reasons that QR factorization is widely used in adaptive applications is that in RRQR the signal information can be effectively updated, making the algorithm suitable to track moving sources.

The real data matrix $G\Psi G^H$ in (25) with dimensions $(N+1)\times(N+1)$ using the QR factorization can be expressed as the product of a real and orthogonal $(N+1)\times(N+1)$ matrix and an $(N+1)\times(N+1)$ rank-revealing upper triangular matrix with real entries. Then the factorization of $G\Psi G^H$ can be written as

$$G\Psi G^H = QR = Q\begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \tag{26}$$

where R_{11} is an $(K\times K)$ matrix, R_{12} is an $K\times(N-K+1)$ matrix and R_{22} is an $(N-K+1)\times(N-K+1)$ matrix, and all real matrices. The QR factorization in (26) is called rank-revealing QR factorization if R_{22} has a small norm. Since R_{22} is very small, the basis of the noise space can be obtained from the above R factor. Let V represent any vector in the null space of R, i.e., $RV=0$. To find the structure of V, we partition V into v_1 with K components and v_2 with *(N-K+1)* components. Then $RV=0$ implies that

$$\begin{bmatrix} R_{11} & R_{12} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \tag{27}$$

so that $R_{11}v_1 + R_{12}v_2 = 0$. Since R_{11} is a non-singular matrix, v_1 can be written in terms of v_2 as follows

$$v_1 = -R_{11}^{-1}R_{12}v_2 \tag{28}$$

Thus V can be written as

$$V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -R_{11}^{-1}R_{12} \\ I_{N-K+1} \end{bmatrix} v_2 \tag{29}$$

To find the basis of the null space of R, we choose any set of $N-K+1$ linearly independent vectors; for example, the columns of the $N-K+1$ dimensional identity matrix. The basis for the null space of the upper triangular matrix R, which is also the null space of $G\Psi G^H$, is therefore given by

$$E_z = \begin{bmatrix} -R_{11}^{-1}R_{12} \\ I_{N-K+1} \end{bmatrix} \tag{30}$$

It is important to observe that the columns of E_Z are not orthonormal in contrast to the null space which can be derived from the SVD or EVD techniques. We now note that the subspace spanned by the columns of E_Z is orthogonal to the subspace spanned by the columns of $B(\theta)$, where the column of $B(\theta)$ contains the information about the AOAs of incident sources. This is similar to the well-known MUSIC algorithm in which the eigenvector of the noise subspace is orthogonal to the steering vector of the signal subspace. To find the AOAs, we search the minimum peaks of $\left\|E_z^H B(\theta)\right\|$. Since the basis of E_Z is not orthonormal we use the orthogonal projection onto this subspace to improve the performance by making the basis of the null space of E_Z orthonormal.

Since we consider the uniform linear array, the proposed method employs the root MUSIC [21] to estimate the AOAs for the incident signals. The proposed unitary root MUSIC converts the power spectrum of the MUSIC algorithm into a polynomial whose roots contain information about the elevation angle θ as

$$F^{-1}(\theta) = B^H(\theta)G^H E_z E_z{}^H G B(\theta) \tag{31}$$

Since the RRQR factorization is for the real-valued matrix instead of the complex-valued matrix in conventional Root MUSIC, then the complexity of the proposed method Unitary Root MUSIC in (31) is about four times lower than the conventional Root MUSIC.

To make the basis of E_Z orthonormal we use the orthogonal projection onto this subspace, which is given by

$$W_o = E_z\left(E_z^H E_z\right)^{-1} E_z^H \tag{32}$$

This implies that [14]

$$W_o B(\theta) = 0 \tag{33}$$

Now equation (31), using orthogonal projection, becomes

$$F_{op}^{-1} = B(\theta)^H G^H W_0 G B(\theta) \tag{34}$$

The roots of the polynomial in (34) can be used to estimate the elevation angle θ_k of the incident signals.

<u>Step 2</u>. Estimation of Azimuth Angle ϕ_k with Estimate $\hat{\theta}_k$

The estimate $\hat{\theta}_k$ obtained in Step 1 will be used for estimation of the azimuth DOA ϕ_k

The proposed orthogonal projection real-valued rank revealing QR factorization will employ the signal vector X received at the ULA in the *x* axis direction and the MUSIC algorithm. By doing this we can avoid the failure problem

in the joint azimuth and elevation angle estimation and pair-matching problem.

The $((2N+1)\times 1)$ signal vector received from the symmetric uniform linear array which is a function of (θ_l, ϕ_l) on x-axis is given by $(x_{-N}, \dots, x_0, \cdots, x_N)^T$. In the proposed method we map the $((2N+1)\times 1)$ vector to the $(N+1)\times(N+1)$ Toeplitz data matrix as follows

$$\mathrm{X} = \begin{bmatrix} x_0 & x_{-1} & x_{-2} & \cdots & x_{-N} \\ x_1 & x_0 & x_{-1} & \cdots & x_{-(N-1)} \\ x_2 & x_1 & x_0 & \cdots & x_{-(N-2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_N & x_{N-1} & x_{N-2} & \cdots & x_0 \end{bmatrix} z \tag{35}$$

The data matrix in (38) can be rewritten in terms of $\Phi^*, S,$ and $C(\theta,\phi)$ as

$$X = \left[C((\theta,\varphi)S \quad C(\theta,\varphi)\Phi_2^*(\theta,\varphi)S \quad \cdots \quad C(\theta,\varphi)\left(\Phi_2^*(\theta,\varphi)\right)^{N-1} S \right] + Q_X$$

$$C(\theta,\phi) = (\mathrm{c}(\theta_1,\phi_1), \ \ \mathrm{c}(\theta_2,\phi_2), \ \ \dots, \ \ \mathrm{c}(\theta_K,\phi_K)) \tag{36}$$

$$(\theta_k,\varphi_k) = \left(1, \ \ (e_k), \ \ \dots, \ \ (e_k)^N\right) \tag{37}$$

$$e_k = \exp\left(-j\frac{2\pi d \sin\theta_k \cos\varphi_k}{\lambda}\right)$$

$$\Phi_2^* = \begin{bmatrix} e_1^* & 0 & \cdots & 0 \\ 0 & e_2^* & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & e_K^* \end{bmatrix} = diag(e_1^* \quad e_2^* \quad \cdots \quad e_K^*) \tag{38}$$

where Q_X is a complex noise variable. Using the unitary transformation we convert the complex data matrix in (36) to real data as follows

$$\Upsilon = X + JX^*J$$

$$G\Upsilon G^H = G\left(X + JX^*J\right)G^H \tag{39}$$

By applying the same rank revealing QR factorization procedure in (26)-(34) on $G\Upsilon G^H$, the null space E_x can be found in the same way. The orthogonal projection onto this subspace is given by

$$Q_o = E_x\left(E_x^H E_x\right)^{-1} E_x^H \tag{40}$$

The azimuth angle estimation $\hat{\phi}$ can be obtained using the estimation $\hat{\theta}$ which is already found in Step 1. By employing the proposed Unitary MUSIC search peak $\hat{\phi}_k$ can be found from the maximum peaks of the following power spectrum as

$$P\left(\hat{\theta}_k,\varphi\right) = \frac{1}{\mathrm{c}\left(\hat{\theta}_k,\varphi\right)^H G^H E_x E_x^H \mathrm{G}\,\mathrm{c}\left(\hat{\theta}_k,\varphi\right)} \tag{41}$$

for source k, $k=1, \dots, K$ where E_x is similar to E_z in (30). Note that the 2-D DOA dimensional search reduces approximately to a 1-D search since the number of sources is very small. In addition, the complexity of the proposed Unitary MUSIC method in (41) is about four times lower than the conventional MUSIC [24].

B- Two-Dimensional DOA Estimation with orthogonal projection and real rank revealing LU factorization

In this section we present a method to obtain the 2-D DOA estimation azimuth and elevation angles from coherent/non-coherent sources using real rank revealing LU factorization. This method is referred to as orthogonal projection real rank revealing LU factorization (OP-RRRLU). Rank revealing LU factorization [23] can reduce the complexity over that of (RRQR) by a factor of two.

The RRRLU factorization of $G\Psi G^H$ can be written as

$$U(YY)_{LU}U^H = LU = L\begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix} \tag{42}$$

where U_{11} is an $(K\times K)$ matrix, U_{12} is an $K\times(N-K+1)$ matrix and U_{22} is an $(N-K+1)\times(N-K+1)$ matrix, and with real entries. Since U_{11} is a non-singular matrix with full rank equal to K and U_{22} has a small norm, the factorization in (42) is referred to as real rank revealing LU factorization. The basis of the noise space can be obtained from U as follows: the upper triangular matrix U represents the null space of Y. Since U_{22} has a very small norm, the upper triangular matrix U can be written as

$$U = [U_{11} \quad U_{12}]W^T \tag{43}$$

where W is a permutation matrix that represents the row and column interchanges. Let q represent any vector in the null space of U, i.e.,

$$Uq = 0 \tag{44}$$

To find the structure of q, we partition q into q_1 with K components and q_2 with $(N\text{-}K+1)$ components. Equation (43) can be written as

$$[U_{11} \quad U_{12}]\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = 0 \ \ \Rightarrow U_{11}q_1 + U_{12}q_2 = 0 \tag{45}$$

From (45) and the fact that U_{11} is full rank with rank K, q_1 can be written as

$$q_1 = -U_{11}^{-1}U_{12}q_2 \tag{46}$$

Using Equations (47-48) vector q can be written as

$$q = \begin{bmatrix} -U_{11}^{-1}U_{12} \\ I_{N-K+1} \end{bmatrix} q_2 \tag{47}$$

To find the basis for the null space of U, we choose any set of $N\text{-}K+1$ linearly independent vectors. The most obvious

choice is the set of linearly independent columns of the identity matrix. A basis for the null space of the upper triangular matrix U, also the null space of Y, is therefore given by

$$\Omega = \begin{bmatrix} -U_{11}^{-1}U_{12} \\ I_{N-K+1} \end{bmatrix} \quad (48)$$

Note that columns of Ω are not orthonormal as the ones provided by SVD or EVD. We observe that the null spaces of (OP-RRRQR) in (30) and the one in (48) for the proposed OP-RRRUL have the same form and dimensions. We use the orthogonal projection onto this subspace to improve the performance by making the basis of the null space of V orthonormal. The columns of the null space of V become orthonmal as follows

$$\Omega_O = \Omega\left(\Omega^H \Omega\right)^{-1} \Omega^H \quad (49)$$

By applying the same procedure used in Section A, the 2-D DOA estimation can be found using the proposed orthogonal projection real rank revealing LU factorization. Note that the difference between the two proposed methods is how to construct the null spaces to estimate the azimuth and elevation angles for the incident sources

III. SIMULATION RESULTS

For simulation, the spacing between the two adjacent elements in the uniform linear array was set to a half wavelength of the incoming signals. Further, one single snapshot per trial and 50 independent trials in total were tested. The performance of the two proposed methods (OP-RRRLU) and (OP-RRRQR) were tested when the sources are coherent.

Figures 2 through 5 show the plots of 2-D DOAs for the two proposed methods (OP-RRRQR) and (OP-RRRLU), respectively. We considered $K=3$ coherent sources, SNR=*10* dB, and *eleven* elements were assumed in each antenna array (the element at the origin is common to both arrays). The elevation and azimuth angles of the two sources were set to (75°, 60°), (90°, 80°), and (110°, 95°) for sources 1, source 2, and source 3 , respectively. Fifty independent trials were performed for each figure. Figures 2 and 3 show the histogram plot for the elevation angle and the power spectrum plot for the azimuth angle, respectively, for the proposed (OP-RRRQR) method. It is observed clearly in Figures 2 and 3 that the proposed algorithm gives accurate 2-D *elevation* and *azimuth* DOA estimations both sources and an exact peak occurred at (75°, 60°), (90°, 80°), and (110°, 95°) in almost all the cases. Figures 4 and 5 show the histogram for the elevation angle and the power spectrum for the azimuth angle using the proposed (OP-RRRLU). From these figures we observe that (OP-RRRLU) gives an accurate estimation for the azimuth and elevation angles for the three sources, and a clear peak occurs at the exact directions. Note that the two proposed methods can estimate the 2-D DOA estimation efficiently. However, the complexity of (OP-RRRLU) is lower than that of (OP-RRRQR) by a factor of two.

IV. CONCLUSIONS

In this paper, we propose two methods for estimating the 2-D direction of arrival, employing the Root MUSIC and MUSIC algorithm for coherent and non-coherent sources. The received data is arranged into a Toeplitz matrix which enables us to perform the estimation using only a single snapshot and detect the incident sources whether they are coherent or not without any spatial smoothing. In addition, the unitary transform successfully employs to transfer the complex data of RRQR and RRLU factorization to real data. Moreover, it does not require pair-matching between elevation and azimuth angle estimation. These advantages make our proposed method suitable for real time implementation.

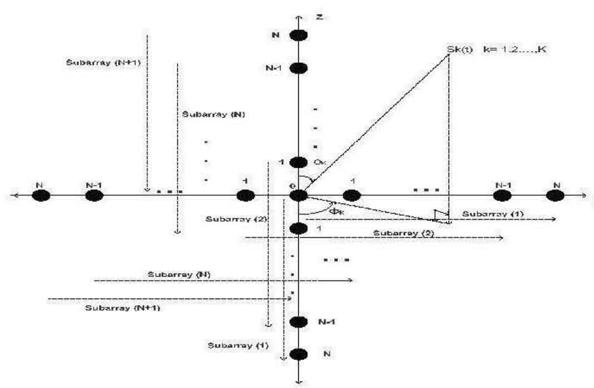

Figure. 1. Block diagram for proposed algorithm for joint 2-D DOA elevation and azimuth angle

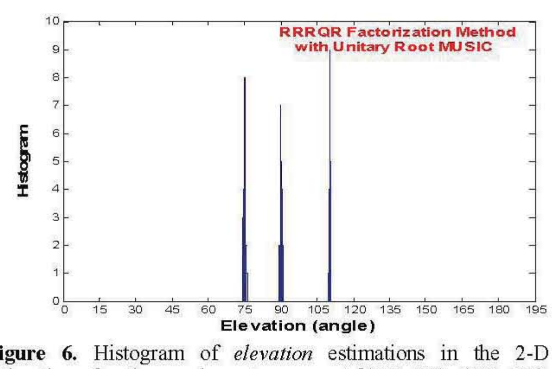

Figure 6. Histogram of *elevation* estimations in the 2-D DOA estimations for *three coherent* sources at [(75°, 60°), (90°, 80°), (110°, 95°)] using the (OP_RRRQR) Method

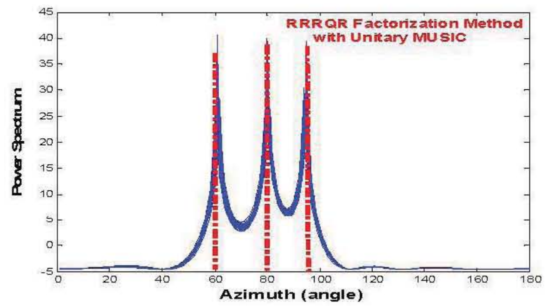

Figure 7. Power spectrum of azimuth estimations in the 2-D DOA estimations for *three coherent* sources at [(75°, 60°), (90°, 80°), (110°, 95°)] using the (OP_RRRQR) Method

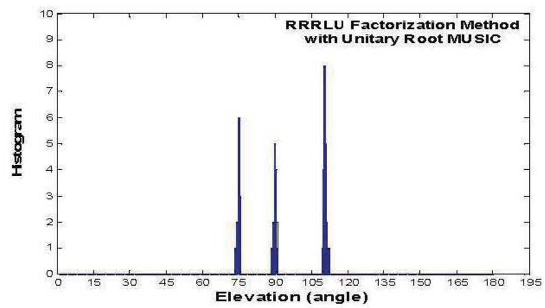

Figure 8. Histogram of *elevation* estimations in the 2-D DOA estimations for *three coherent* sources at [(75°, 60°), (90°, 80°), (110°, 95°)] using the (OP_RRRLU) Method

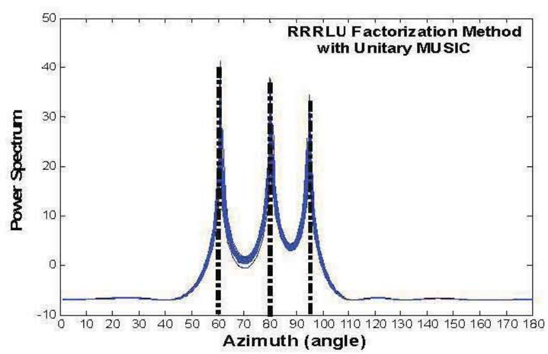

Figure 9. Power spectrum of azimuth estimations in the 2-D DOA estimations for *three coherent* sources at [(75°, 60°), (90°, 80°), (110°, 95°)] using the (OP_RRRLU) Method

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